Tuned Mass Damper of the Taipei 101 Tower

City of Taipei

Facts:

- Capital of Taiwan
- Population size of 2.7 million (2015)
- Density – 10000/km²
- 40th most populous urban area in the world
- Ranked 69 in the world for most 150m+ completed buildings (13 completed)

Seismic Activity in Taiwan

- Taiwan is located on the cusp of the Pacific “ring of fire”, making it very susceptible to frequent seismic shocks.
- 42 active faults have been identified by geologists on the island of Taiwan
- The reason for most of the earthquakes in Taiwan is due to the convergence of the Philippines Sea Plate and Eurasian Plate found to the east of the island.
- Between 1901-2000, 91 major earthquakes (magnitude of 6 and above on the Richter scale) were recorded.
- 1999 Jiji earthquake – one of the deadliest in the history of Taiwan
Structural Development in Taiwan

- Before 1980, Taiwan was a relatively poor country, having few professional architects and lacked the understanding of earthquake safety.
- Designer and developers cut corners and used connections in local governments to illegally acquire permits.
- Many modern buildings in Taiwan today are constructed with earthquake safety in mind, including Taipei 101, which has to cope with the dual challenges of being flexible enough to withstand earthquakes, yet rigid enough to resist incoming winds.

Taipei 101

Facts:
- Constructed in 2004
- 509 meters tall
- 101 floors
- Tallest building in the world from 2004 to 2010
- Sits 660 feet from a major fault line
- Taipei world financial center
- LEED (Leadership in Energy and Environmental Design) platinum certification for being largest green building in the world

Tuned Mass Damper

Facts:
- World's largest tuned mass damper
- Structure introduced to building to withstand winds up to 216 km/hr, typhoons and major earthquakes.
- 728-ton (728 000 kg) structure
- 18 ft. diameter steel sphere
- 8 viscous dampers are attached to the sphere, acting as shock absorbers
- System is capable of reducing wind vibrations by up to 40%
- Point of tourism
- Cost of 4 million Dollars (USD)
- Designed my Motioneering

How does a Tuned Mass Damper Work?

- A TMD is capable of reducing the vibrational amplitude, consequent of the external force. It does so by absorbing kinetic energy from the system, which in this case is the swaying motion of a tall building due to the wind or an earthquake.
- The frequencies and amplitudes of the TMD and the structure should be tuned (i.e. set to be nearly equal) in order for that when a force is applied on the structure, the TMD can create an equal and opposing force. This creates a cancellation of forces and keeps the horizontal displacement of the structure to approximately zero.
The mechanical system $m_1$, $k_1$, $c_1$ is the oscillator to be damped (i.e. Taipei 101) and $m_2$, $k_2$, $c_2$ is the damping oscillator. (i.e. Tuned Mass Damper). We will assume that the external force being applied on the main structure ($m_1$) is sinusoidal. This external force can come from strong winds, typhoons or earthquakes. The $\omega$ is the vibrational frequency, $t$ is the time, and $p_0$ is the magnitude of the sinusoidal excitation.

Before we begin, we need to assume that the system is undamped ($c_1=c_2=0$) to make calculations easier. The Schematics can then be represented by the following equations of motions:

$$m_1 \ddot{x}_1 + (k_1 + k_2)x_1 - k_2x_2 = p_0 \cos(\omega t)$$

$$m_2 \ddot{x}_2 - k_2x_1 + k_2x_2 = 0$$

To solve this 2$^{\text{nd}}$ Order Differential Equations, we need to first define a steady state solution.

$$x = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \cos(\omega t)$$

Since there is no damping in the system (our assumption earlier), there is no phase shift between the steady state solution and the external force equation. They are of the same harmonic nature.

By plugging in the steady state solution into our equations of motions, we get an expression of the form:

$$-\omega^2 m X + kX = p_0$$

We then introduce the following variables to the equations:

a. Natural frequency of the main system $\omega_n = \sqrt{\frac{k_1}{m_1}}$

b. Natural frequency of the TMD $\omega_a = \sqrt{\frac{k_2}{m_2}}$

c. Static Displacement $X_{st} = \frac{p_0}{k_1}$
d. Mass Ratio $\mu = \frac{m_2}{m_1}$

- By doing this, we can determine the values of $X_1$ and $X_2$.

$$X_1 = \frac{\left(1 - \left(\frac{\omega}{\omega_a}\right)^2\right) X_{ST}}{\left(1 + \mu \left(\frac{\omega_a}{\omega_n}\right)^2 - \left(\frac{\omega}{\omega_n}\right)^2\right)\left(1 - \left(\frac{\omega}{\omega_a}\right)^2\right) - \mu \left(\frac{\omega_a}{\omega_n}\right)^2}$$

$$X_2 = \frac{X_{ST}}{\left(1 + \mu \left(\frac{\omega_a}{\omega_n}\right)^2 - \left(\frac{\omega}{\omega_n}\right)^2\right)\left(1 - \left(\frac{\omega}{\omega_a}\right)^2\right) - \mu \left(\frac{\omega_a}{\omega_n}\right)^2}$$

- The point of all this is to make the amplitude $X_1 = 0$, as this is the motion (swaying) of the main structure we want to minimize. To do this, we need to make the numerator of $X_1$ equal to zero. By doing so we can determine the following:

$$X_2 = -\frac{p_0}{k_2}$$

- This means if spring number two $(k_2)$ applies a force of $-p_0$, it can cancel the external force applied to the main structure.

**What is Resonance?**

- Resonance is a phenomenon where an external oscillatory force is applied to a system, causing it to vibrate at one of its natural frequencies. Vibrating at a natural frequency can result in very large, dangerous amplitudes of displacement.

- Common engineering practice always avoids making a system vibrate at resonance.

- Critical frequency ratios of the system depend solely on the mass ratio selected. Typical engineering designs have a mass ratio between 0.05 and 0.25.

**Conclusion?**

- TMD’s are an efficient way of attenuating large amplitudes with a relatively small system. The weight of the TMD system is only 1% of the total weight of the Taipei 101 tower! They allow engineers the freedom to design the system with respect to the necessary constraints (i.e. size, weight, external forces).